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INNOVATIVE METHODS AND TECHNIQUES OF TEACHING MATHEMATICAL MODELING AND RESEARCH INTO THE MANUFACTURING PROCESS MANAGEMENT

Graduates working toward their technical, natural sciences, or economic degrees should be prepared for the reality of project management and rational organization of modern manufacturing demanding synthesis and profound knowledge in various fields of science.

Ensuring the coordinated functioning and interaction of the groups of people and machine complexes requires a careful study of methods and approaches to effective problem-solving. The scientific direction that provides quantitative basis for decision-making is called "operations research". The stages of operations research are the following: generating a mathematical model of the phenomenon (operation) \rightarrow analysis of the model \rightarrow implementation of the research results.

To generate models and solve them, certain statistical and mathematical methods and systems of computer mathematics are required (SCM).

The criterion for the existence of this applied science is a comprehensive approach to the tasks set, cooperation and agreement of experts in various fields (economists-mathematicians, engineers, operators, ecologists, agronomists, experts, accountants, managers). That is why everyone who is involved in manufacturing management should be familiar with the operations research methods.

At the same time, students' perception and understanding of the disciplines, the essence and subject matter of which are interrelated (mathematical modeling of systems and processes, mathematical methods and models, operations research, mathematical programming, methods of optimization, etc.) still present a problem.

One of the topical tasks of math education and higher educational is a well grounded, available, and profound presentation of methods and techniques of the afore-mentioned disciplines, the subject of study of which are the models, optimization tasks and their solutions by means of SCM.

Methods and models of the operations research are discussed in many scientific works and textbooks.

Worth mentioning is a book by A. Kofman (1966), which discusses the branches of this discipline for the experts of "non-mathematical" majors (administrators, engineers, and planners) in a popular, accessible form. As an engineer and mathematician, the scholar pointed out that, "This book could have had the name *"Applied mathematics of manufacturing process management"*, but it would scare away those economists who have not yet got used to considering mathematics as the necessary working tool" [2, p. 25]. The author also shows the use of computers in problem-solving: description of operations on a calculating machine *"Gamma"* with a storage device on a magnetic drum.

A book written by Hemdi A. Takha, first was published in 1971, bears no excessive academism as well [4]. The sixth updated edition contains numerous examples and exercises (about 1000) as a means of presentation of basic ideas and principles that underlie the various methods of the theory of operations research. The TORA software and the programming language SIMMET II are part of this scientific work.

However, the analysis of modern educational literature on mathematical modeling, mathematical programming, operations research (e.g., by such authors as V. Vitlins'ky, S. Nakonechny, T. Tereshchenko (2001), O. Ul'yanchenko (2002), V. Kihel' (2003), M. Krass, B. Chuprynov (2006), O. Kundysheva (2007), F. Vashchuk, O. Laver, N. Shumilo (2008), M. Kuchma (2011.), et al) reveals the lack of attention to both the potential and methods of *SCM* for solving specific problems, the prospect of obtaining the results using the computer is mentioned only sporadically.

Development of laboratory sessions on the methods of forecasting and optimization are presented exclusively for the tabular processor *MS Excel*, such as in the manual by T. I. Makarenko (2005). However, students should be taught to solve real-life problems using several systems of computer mathematics, which increases the probability of obtaining correct results and allows for the analysis of the obtained solutions.

Students of mathematical majors are afforded the opportunity to review the main features and characteristics of *MathCAD*, *Maple, MATLAB, Mathematica systems* and MS Excel Solver Add-in for solving various types of optimization problems in the works by Yu. Tryus [3].

The purpose of the article is to reveal certain aspects of teaching methods of mathematical modeling and solving practical problems based on the matrix theory using the system of computer mathematics *MathCAD*.

It is important to note that students of economic, technical specialties should be given an opportunity to solve such problems not only during mathematical modeling or operations research classes, but within the course of advanced mathematics (or mathematics for economists) when studying linear algebra as well, which will become the introduction to economic-mathematical methods [1].

Matrix algebra is one of the main methods of solution of many business problems. Especially useful it is for the development and use of databases: when working with them almost all data are stored and processed in a matrix form. With the systems of linear equations and matrices, the following tasks of *macroeconomics* can be solved: pricing in input-output studies, development of the balance model of multi-branch economy, export-import models, and international trade models.

The main purpose of economics is the rational functioning of business entities, namely, satisfaction of needs and efficient work making use of limited resources. In *microeconomics*, linear programming (LP), methods of which are used in forecasting, planning and organization of production processes, as well as in the financial sector, is the most prominent and effective. LP algorithms calculations are usually bulky and uniform, but computer technology makes their implementation quite exciting. To do

this, a set of constraints (a system of equations or inequations) of models of optimization problems is also given in the form of matrices.

To solve problems, a universal mathematical software *MathCAD*, recognized as one of the top for scientific, technical, engineering, and economic problemsolving, was selected. To enter formulas in other *SCM* as well as in spreadsheets, a very complex syntax is used, while *MathCAD's* functions and procedures are easy to learn, and the composition of its worksheets resembles that done *on paper with a pencil in hand*.

• Let us consider the problem of production planning by means of the balance models of multi-branch economy.

Preface. Macroeconomics of an enterprise functioning requires the balance between different branches. Each branch, on the one hand, is a manufacturer; on the other hand, it is a consumer of products of other industries. Thus, input-output calculations are quite difficult. For the first time (1936), this problem was formulated as a mathematical model by a famous American economist V. Leontief, who made an attempt to analyze the causes of economic depression in the U.S. (1929-1932). This model is based on matrix algebra and uses the matrix analysis .

Statement of problem 1. The table features the balance among five branches of industry for a certain period of time.

N⁰	Industry	Consumption					Final	Gross
		1	2	3	4	5	output	output
1.	Mining and processing of hydrocarbons	3	15	10	7	5	50	100
2.	Power economy	7	3	8	10	11	60	100
3.	Machine building	10	5	6	10	8	5	50
4.	Machine-tool building	16	12	19	15	18	10	100
5.	Automobile industry	5	5	7	10	6	15	50

Determine the performance of coefficient matrix of direct costs. Find the volume growth of gross output of each product (percentage), if the final consumption by industry increases by 50%, 10%, 70%, 100%, and 160%, respectively.

Problem-solving instructions

1 Enter the vectors of gross output **X1** and final consumption **Y1**, matrix of coefficients of direct costs **A**, and the vector of potential percentage change of the

final product ΔY . Then, a new vector of consumption of the final product must be Y2 $= \mathbf{Y}\mathbf{1} + \mathbf{Y}\mathbf{1} \cdot \mathbf{\Delta}\mathbf{Y}.$

2 Check the productivity of matrix **A**. Set the number of columns **t** of matrix A, that is determined with the integrated **cols** (A). The function *identity* (t) creates a unit matrix of t order. Obtain an integral part of the inverse matrix $(E-A)^{-1}$.

To determine the productivity of the matrix A, the amount of r_m elements of its columns should be calculated (matrix $A \ge 0$ is *productive* if the sum of the elements of its any column does not exceed 1). This leads us to the conclusion that, for the given balance sheet, all the industries are cost efficient, because all of the re_m responses to the question whether $r_m \leq 1$ or $m = 1 \dots t$, are positive.

3 Find the components of a new vector of gross output $X2 = (E-A) - 1 \cdot Y2$, and with the formula $X2 = X1 + X1 \cdot \Delta X$ get the growth ΔX of the gross output in percentage: $\Delta X = (-1) \cdot 100\%$.

MathCAD-solution:

$$\begin{aligned} \text{ORIGIN} &:= 1 \\ \text{X}_{1} := \begin{pmatrix} 100\\ 100\\ 50\\ 100\\ 50 \end{pmatrix} \quad \text{Y}_{1} := \begin{pmatrix} 50\\ 60\\ 5\\ 10\\ 15 \end{pmatrix} \quad \text{A} := \begin{pmatrix} 0.03 & 0.15 & 0.2 & 0.07 & 0.01\\ 0.07 & 0.03 & 0.16 & 0.1 & 0.22\\ 0.1 & 0.05 & 0.12 & 0.1 & 0.16\\ 0.16 & 0.12 & 0.38 & 0.15 & 0.36\\ 0.05 & 0.05 & 0.14 & 0.1 & 0.12 \end{pmatrix} \quad \Delta \text{Y} := \begin{pmatrix} 0.5 \\ 0.1 \\ 0.7 \\ 1\\ 1.6 \end{pmatrix} \\ \text{Y}_{2} := \text{Y}_{1} + \overline{(\text{Y}_{1} \cdot \Delta \text{Y})} \qquad \text{Y}_{2}^{\text{T}} = (75 & 66 & 8.5 & 20 & 39) \\ \text{t} := \text{cols}(\text{A}) \qquad \text{m} := 1 \dots \text{t} \qquad \text{E} := \text{identity (t)} \\ \text{r}_{\text{m}} := \sum \text{A}^{\sqrt{\text{m}}} \qquad \text{re}_{\text{m}} := \text{if}(\text{r}_{\text{m}} \le 1, "\text{yes"}", "\text{no"}) \end{aligned}$$

 $X_2 := (E - A)^{-1} \cdot Y_2$ $X_2^T = (117.075 \ 117.02 \ 58.199 \ 122.317 \ 80.778)$

"yes"

$$\Delta \mathbf{X} := \left(\overrightarrow{\left(\frac{\mathbf{X}_2}{\mathbf{X}_1} \right)} - 1 \right) \cdot 100$$

Thus, in order to ensure the desired growth in the final product, it is necessary to increase the corresponding gross output: mining and processing of hydrocarbons by 17,1%, power output by 17%, machine building by 16,4%, machine-tool building by 22,3%, automobile industry output by 61,6% compared to the original data.

Students get 6 variants of homework: solve the problem for three industries in a written form (1-2-3, 2-3-4, 3-4-5, 1-3-4, 1-4-5, 2-4-5) by Gauss-Jordan method and verify the answer using the computer.

• Let us consider the problem on finding the optimal cropping pattern.

Preface. At the micro level, manufacturing management presupposes having to make many decisions. All of them require an analytical substantiation: the same costs can produce different economic effects depending on the decisions taken. It is necessary to prepare and consider various solution options and determine the optimal one. Linear programming (LP) is a method of mathematical modeling designed to optimize the use of limited resources. It was developed in 1939, by the Soviet scientist L. Kantorovich. Among LP tasks are the following: feed ration development, resources management and appraisal, optimal machine loading, optimal output plan, transportation problem.

Statement of problem 2. An agricultural farm owns two fields of 80 ha and 40 ha, which differ by the location and soil characteristics. The farm intends to grow carrots and cabbage. It is known, that the yield of carrots is 450 centner/ha on the first field and 500 centner/ha on the second field, cabbage is 300 and 350 centner/ha, respectively. To get the desired yield, phosphorus and potassium fertilizers will be used. Fertilizing rates are the following: phosphorus for carrots - 60 and 80 kg/ha, for cabbage - 100 and 120 kg/ha, potassium for carrots - 90 and 100 kg/ha, for cabbage - 100 and 140 kg/ha for the first and the second fields, respectively. The stock of fertilizers is limited and totals 8 tons of phosphorus and 10 tons of potassium. An approximate purchase price for 1 centner of carrots is 10 MU, for cabbage – 20 MU.

Determine the optimal cropping pattern in the farm, which will assure the highest revenue from the sale of the vegetables.

Solution

1) *Generating a mathematical model*. The land allocated for carrots is x_1 ha in field 1 and x_2 in field 2, for cabbage - x_3 and x_4 ha in fields 1 and 2, respectively. The revenue from the sale, for instance, of carrots on field 1:

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450 centner/ha • 10 MU/centner • \cdot x_1 ha = 4500x_1 MU
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Then, the target function, which should maximize the total revenue from the sale of agricultural products, is as follows: $f(x) = 4.5x_1 + 5x_2 + 6x_3 + 7x_4 \rightarrow max$.

Constraints on the land use and fertilizers stock:

$$\begin{cases} x_1 + x_3 \le 80 \\ x_2 + x_4 \le 40 \\ 60x_1 + 80x_2 + 100x_3 + 120x_4 \le 8000 \\ 90x_1 + 100x_2 + 100x_3 + 140x_4 \le 10000 \\ X_i \ge 0, i = 1-4 \end{cases}$$

2) Finding the optimal solution in the MathCAD. The algorithm:

- Set the matrices *B*, *C*, *F*, and *K*, namely, enter the initial data of yields, purchase prices for agricultural produce, fertilizers rates F and C, and fertilizers stock deficiencies - *OD*.

- Set the starting point in numbering of rows and columns of matrices (ORIGIN:=1), initial values of variables, form a supporting matrix A for matrix notation of inequations, and the target function f(X). For multiplying the elements of matrices B and C, do the vectoring of $B \cdot C$.

- Form a unit for solving the problem that begins with the word "Given". An integrated function Maximize(f,X), which gives f(X) maximum, should complete the unit.

- Get the optimum plan Y=... and the optimal value of the target function f(Y)=... A search for the minimum or maximum can be done using a number of methods in MathCAD. It is recommended to test and compare alternative solutions for each method.

MathCAD-document:

$$\begin{split} \mathbf{B} &:= (450 \ 500 \ 300 \ 350) \quad \mathbf{F} := (60 \ 80 \ 100 \ 120) \\ \mathbf{C} &:= (10 \ 10 \ 20 \ 20) \qquad \mathbf{K} := (90 \ 100 \ 100 \ 140) \qquad \mathbf{OD} := (8000 \ 10000) \\ \mathbf{ORIGIN} := 1 \quad \mathbf{i} := 1 \dots 4 \quad \mathbf{X}_{\mathbf{i}} := 0 \qquad \mathbf{f}(\mathbf{X}) := \overrightarrow{(\mathbf{B} \cdot \mathbf{C})} \cdot \mathbf{X} \qquad \mathbf{A} := \operatorname{stack}(\mathbf{F}, \mathbf{K}) \\ \overrightarrow{(\mathbf{B} \cdot \mathbf{C})} &= \left(4.5 \times 10^3 \ 5 \times 10^3 \ 6 \times 10^3 \ 7 \times 10^3\right) \bullet \\ \mathbf{Given} \qquad \mathbf{A} \cdot \mathbf{X} \le \mathbf{OD}^{\mathrm{T}} \qquad \mathbf{X} \ge 0 \qquad \mathbf{X}_1 + \mathbf{X}_3 \le 80 \qquad \mathbf{X}_2 + \mathbf{X}_4 \le 40 \\ \mathbf{Y} := \operatorname{Maximize}(\mathbf{f}, \mathbf{X}) \qquad \mathbf{Y}^{\mathrm{T}} = (50 \ 25 \ 30 \ 0) \bullet \qquad \mathbf{f}(\mathbf{Y}) = 5.3 \times 10^5 \bullet \end{split}$$

3) Conclusion based on the obtained solution. The optimal cropping pattern in the farm is the following: carrots - 50 ha of field 1 and 25 ha of field 2, cabbage - 30 ha of field 1, while its cultivation is not recommended on field 2. In this case, the sales of agricultural produce give the largest profit - 530000 MU At the same time, 15 ha remain unused on field 2.

To motivate students and consolidate their skills, the main task is supplemented with additional conditions, whereas the work at the lesson can be accompanied by a business game. The "manager" selected from among the students divides the subordinates into 4 groups giving each of the groups a separate task. The farm's leadership is interested in the research into and analysis of the situations given below; it is expected that an optimal solution for each of them will be found.

1. The farm can additionally buy 1 ton of phosphorus and 1 ton of potassium fertilizers at the price of 100 MU/kg and 80 MU/kg, respectively. Is it profitable? How may the copping pattern and the revenue change in this situation?

2. The farm received an additional order for at least 11.750 centners of cabbage. Is this offer beneficial? How will the cropping pattern and the farm's revenue change in this situation?

3. The farm obtained the information that the cabbage purchase price will increase to 25 MU/centner. The farm's leadership is interested whether they have to review the cropping pattern and how it can affect the revenue.

4. It is planned also to use the first field for beets for the coming year. The standard fertilizer rate is as follows: phosphorus - 70 kg/ha, potassium - 90 kg/ha. The expected yield of beets on the first field is 500 centner/ha, and the purchase price

for 120 MU/kg. Determine how can the addition of one more culture affect the cropping pattern.

The groups correct mathematical models, find and analyze the optimal solution, prepare reports and submit them to the "manager".

Correction of mathematical model of the main problem

1.
$$f(x) = 4.5x_1 + 5x_2 + 6x_3 + 7x_4 \rightarrow max \text{ minus } 180000$$

$$\begin{cases} x_1 + x_3 \le 80 & \text{OD} := (9000 \ 11000) \\ Y := \text{ Maximize}(f, X) \\ 60x_1 + 80x_2 + 100x_3 + 120x_4 \le 9000 \\ 90x_1 + 100x_2 + 100x_3 + 140x_4 \le 11000 \\ x_i \ge 0, i=1..4 \end{cases}$$

$$\begin{cases} Y^T = (43.75 \ 34.375 \ 36.25 \ 0) \\ f(Y) = 5.862 \times 10^5 \\ f(Y) - 180000 = 4.063 \times 10^5 \end{cases}$$

Conclusion 1. If 1 ton of fertilizers is additionally purchased, the optimal cropping pattern will be as follows: carrots - 44 ha on field 1 and 34 ha on field 2, cabbage - 36 ha on field 1. On the second field the cultivation of cabbage is still not expected, and 4 ha remain unused. The maximum revenue from the sale of is 586200 MU. But 180000 MU spent on fertilizers, decreases the income to 406300 MU.

2.
$$f(x) = 4.5x_1 + 5x_2 + 6x_3 + 7x_4 \rightarrow max$$

 $x_1 + x_3 \le 80$
 $x_2 + x_4 \le 40$
 $60x_1 + 80x_2 + 100x_3 + 120x_4 \le 8000$
 $90x_1 + 100x_2 + 100x_3 + 140x_4 \le 10000$
 $300x_3 + 350x_4 \ge 11750$
 $x_i \ge 0, i = 1..4$
MathCAD-document 2
 $300 \cdot X_3 + 350 \cdot X_4 \ge 11750$
 $Y^T = (52.5 \ 11.25 \ 27.5 \ 10) \bullet$
 $f(Y) = 5.275 \times 10^5 \bullet$

Conclusion 2. If the farm receives an additional purchase order for no less than 11.750 kg of cabbage, then the optimal cropping pattern will be as follows: carrots - 52,5 ha on field 1 and 11 ha on field 2, cabbage - 27,5 ha on field 1 and 10 ha on field 2. The first field is used to the full capacity; approximately 19 ha are used on the second field. The maximum total revenue is 527.500 MU.

 $3. f(x) = 4.5x_1 + 5x_2 + 7.5x_3 + 8.75x_4 \rightarrow max \qquad MathCAD-document 3:$ $x_1 + x_3 \le 80$ $x_2 + x_4 \le 40$ $60x_1 + 80x_2 + 100x_3 + 120x_4 \le 8000$ $90x_1 + 100x_2 + 100x_3 + 140x_4 \le 10000$ $Y^{T} = (0 \ 0 \ 80 \ 0) \bullet$

 $x_i \ge 0, i = 1..4$

$$f(Y) = 6 \times 10^5 \, \bullet$$

Conclusion 3. If cabbage purchase price reaches 25 MU/centner, the cultivation of this crop is only profitable on 80 ha of field 1; field 2 should remain unused. The maximum revenue of the farm is 600.000 MU.

4. $f(x) = 4.5x_1 + 5x_2 + 6x_3 + 7x_4 + 60x_5 \rightarrow max$ $\begin{cases} x_1 + x_3 \le 80 \\ x_2 + x_4 \le 40 \\ 60x_1 + 80x_2 + 100x_3 + 120x_4 + 70x_5 \le 8000 \\ 90x_1 + 100x_2 + 100x_3 + 140x_4 + 90x_5 \le 10000 \\ x_i \ge 0, i = 1..5 \end{cases}$

MathCAD-document 4:

Conclusion 4. If beets cultivation is planned for field 1, then the optimal cropping pattern changes significantly: on 80 ha of field 1, it is profitable to grow only beets, whereas on 20 ha of field 2 - cabbage. It is not planned to grow carrots, and 20 ha of field 2 remain empty. The maximum revenue is 4.940.000 MU.

There is an alternative solution, which is calculated via *MathCAD* using a different method: $Y^{T} = (0 \ 28 \ 0 \ 0 \ 80)$ $f(Y) = 4.94 \times 10^{6}$ - the revenue will be exactly the same if field 1 is planted with beets, and 28 ha of field 2 are planted with carrots.

Summarizing the work of students during the class session, we should emphasize, that the obtained solutions are a tool for decision-making, suggestions on how to succeed in business, which are recommended for implementation after a thorough experience-based analysis.

Conclusions. Great strategists and businessmen have been, pretty much, the people of intuition. But the times, when a manager could draw all the data for decision-making from his/her own head, have passed. An evolution has occurred: the

slogan "from experience to intuition" have been substituted with the rule "from information to conclusion" or with the pattern "statistics - mathematical methods - inference".

Misunderstanding, fear, ignorance, and, hence, the inhibition of introduction of the economic and mathematical methods into economic practices is still the case. The reason for that is the lack of information shared with students about the potential and prospects of the methods of advanced mathematics, its computer tools for mathematical modeling and operations research for their future careers.

It is necessary to inform students in an accessible form that this scientific area, despite being deeply-rooted in mathematics, is able to avoid complex mathematical calculations. It is necessary to understand the nature of its typical problems, methods and algorithms of their solution, whereas an additional tool in finding the solution should be the computer software.

The use of SCM in the educational process provides a real opportunity to the future experts to get used to the work according to the following order: *formalization* of the initial problem \rightarrow generation of a mathematical model \rightarrow selection of a mathematical method and algorithm for solving the model \rightarrow implementation of the solution using the computer \rightarrow verifying the model \rightarrow decision-making and implementation of the solution.

We can distinguish the following prospective lines of the further research in this area:

- development of the recommendations for the use of SCM by the students of "non-mathematical" majors, such as: "Ecology", "Agronomy", "Forestry", "Geodesy and Land Use", "Economics," "Management";

- development and improvement of computer-oriented methodological teaching/learning systems for the following majors: "Mathematical modeling of systems and processes", "Mathematical methods and models", "Operations research", "Mathematical programming", "Methods of optimization";

- development of electronic textbooks and distance learning courses for the mentioned subjects.

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Korniychuk O. E. Innovative Methods and Techniques of Teaching Mathematical Modeling and Research into the Manufacturing Process Management

The article presents the implementation of some methods of mathematical modeling and operations research in the educational process, namely the solution of real-life problems based on the matrix theory applying the system of computer mathematics *Mathcad*. It describes the methods of generating and analyzing mathematical models for a manufacturing planning problem utilizing the balance model of multi-branch economy, as well as problems of finding the optimal cropping pattern. The algorithms and problem-solving techniques of these models are set forth in the package Mathcad.

Key words: operations research, optimization problem, model, mathematical modeling, decision-making, systems of computer mathematics.

Корнійчук О. Е. Новітні методи і прийоми навчання математичного моделювання та дослідження організації виробництва

У статті представлено реалізацію деяких методів математичного моделювання та дослідження операцій в процесі їх вивчення, а саме при розв'язуванні реальних задач на основі теорії матриць з використанням системи комп'ютерної математики *Mathcad*. Розкрито методику побудови та аналізу математичних моделей для задачі з планування виробництва за допомогою балансової моделі багатогалузевої економіки та задач на знаходження оптимальної структури використання земельних площ. Подано алгоритми та технології розв'язання цих моделей у пакеті *Mathcad*.

Ключові слова: дослідження операцій, задача оптимізації, модель, математичне моделювання, прийняття рішення, системи комп'ютерної математики.

Корнейчук Е. Э. Новейшие методы и приемы обучения математического моделирования и исследования организации производства

В статье представлена реализация некоторых методов математического моделирования и исследования операций в процессе их изучения, а именно при решении реальных задач на основе теории матриц с использованием системы компьютерной математики *Mathcad*. Раскрыта методика построения и анализа математических моделей для задачи планирования производства с помощью балансовой модели многоотраслевой экономики, а также задач на нахождение оптимальной структуры использования земельных площадей. Изложены алгоритмы и технологии решения этих моделей в пакете *Mathcad*.

Ключевые слова: исследование операций, задача оптимизации, модель, математическое моделирование, принятие решения, системы компьютерной математики.

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