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# ANALYTICAL GEOMETRY TESTING AND THE WAYS OF ITS PRACTICAL IMPLEMENTATION

The paper focuses on the principles of composing tests in geometry for students working toward their degree as teachers of Physics and Mathematics.

The changes in the approaches to teaching/learning and instructional assessment practices in modern Ukrainian school resulted in the advancement of testing as the dominant way of measuring learning outcomes. Tests are used to assess students' achievement in all school subjects, including mathematics. Testing at school as a means of assessment in various disciplines is discussed in the works by L. Buryak, A. Masalitina, D. Raven, et al.

Despite the availability of a vast amount of didactic material (on test preparation and testing procedure), computer software (developed for testing at various stages at which mathematics is studied), on-line tests (for training for the external testing in mathematics), in recent years school graduates have demonstrated poor results: the results of the External Independent Assessment (EIA) suggest that more than a half of all the students have scored less than 150 points out of 200, and about 10% of them have not passed the lower threshold of 124 points. Besides, the number of applicants choosing mathematics as their major in higher educational institutions is quite low. These are the signs of the decline of mathematical education resulting from the low interest of pupils and students in this subject matter. It should be remembered that these same students will soon come to secondary schools as teachers of mathematics to train pupils for the External Independent Assessment. The outcomes of these future instructional assessments are directly dependant on the students' today's preparation and quality of their knowledge. Thus, the need for a more thorough training of prospective teachers to meet the realities of the modern school is becoming obvious. We should prepare our students to teach mathematical disciplines in such a way, so as not to upset the balance between teaching mathematics and "training for the test". Currently, there are numerous advancements in the area of assessing learning outcomes in mathematics, but almost all of them deal with mathematics for secondary school, but not for higher educational establishments. Available research into instructional assessment describes the techniques in general, without reference to specific subjects, namely higher school mathematics. The authors fail to adequately address the specific features and uniqueness of this discipline; they often avoid the interpretation of the implementation of pedagogical techniques aimed at content knowledge assessment on the basis of the concrete learning material. It should also be noted that the vast majority of the existing tests for students ignore the correlation between secondary school and university curricula. It is of critical importance to ensure constant transition from school to university course materials and, then, back again to the secondary school curriculum in the mathematics teachers' professional training. The afore-mentioned speaks clearly to the fact that there is a need to improve mathematical content knowledge assessment techniques for the students of higher educational institutions, in particular, for the prospective teachers of mathematics.

We agree that, under the present-day conditions, the best solution is to create a testing system that "... shall be connected in terms of techniques utilized with the unified system of assessment of school graduates and certification of professionals" [2]. In other words, in addition to its control function, it should serve as an instrument for implementing the systemic approach to training. From this point of view, testing offers a number of advantages over the traditional forms of assessment, such as:

1) higher objectivity of assessment compared to traditional methods;
2) scoring with higher differentiation;

3) facilitating the implementation of the principles of assessment (requirements) (i.e., planned, consistent, and systemic nature of assessment, efficiency, appropriateness in terms of scope, format, and difficulty, objectivity, and transparency [3]) simultaneously for a large number of students.

The use of computer technology adds to the list of these benefits the following:

4) possibility to create a large number of tests for the same field;

5) possibility to combine text and images, which is of particular importance when studying mathematics;

6) possibility to use theoretical foundation as a prompt;

7) automatization of the chain "assessment – reviewing – grading – reporting";

8) control over the testing sequence (for example, return to the previous level);

9) creation of the environment for learning through play, etc.

Computer-based testing facilitates the preparation of students' status reports. It affords the visualization (e.g., in the form of a chart) of the ranked results for the whole group and separately for each student. This approach allows designating the student's score as a measure of his/her growth, allows the student to see the recognition of the fruits of his/her labor from the point of view of an outside observer. Thus, the grade is used to analyze achievements and demonstrates the dynamics of growth.

The development of the system of testing is a complex and creative process, which should result in a product, brought to the level of an instrument, applicable at any stage of the learning process and apt to verify the conformity of the prospective teachers of mathematics to the educational qualification requirements.

Tests are usually defined as "...the system of tasks of a specific format, scope, increasing difficulty that allows to assess the structure and to measure the level of knowledge and skills" [4, p. 15]. On the basis of this definition and using analytical geometry as a concrete learning material, we offer the testing structure that qualitatively assesses the readiness of prospective mathematics teachers for the work at school. It is well known that the tests differ in style (closed-ended, open-ended, matching tests, correct succession tests [4]) and the level of difficulty. Both criteria are taken into consideration in this research.

In order to facilitate the objectivity of knowledge assessment and to maintain the continuity of the school and university education, the following levels of difficulty have been defined:

1) the level of elementary notions given in the form of classical terms;

2) the level of elementary notions presented with the help of "new" terms;

3) the basic level of a higher educational institution; and

4) advanced level.

The first two levels are aimed at testing the knowledge of school geometry course, whereas the third and the fourth are tailored for the standardized curriculum of higher educational institutions. The students' progression from one level to another is viewed as the indication of the acquisition by them of the content knowledge, as well as their preparedness to teach this content in secondary schools. On the other hand, it should be remembered that such differentiation is quite relative and depends on the specific task. The tasks of the first through third levels are close-ended, or multiple-choice, questions, including matching and correct sequence items, while at the fourth level open-ended questions to be assessed by the instructor are used.

The implementation of this structure can be achieved in a number of ways. For instance:

*1. Comprehensive use of the properties of the same concept, studied at different stages of instruction.*

For example, the concept of "vector" is first introduced in secondary school, where its basic properties are studied as well. The same concept, but at a more advanced level, also appears in the course of analytical geometry. Therefore, it is advisable to address it at all four testing levels.

For example:

**The first level:**

Which of the vectors shown on Fig. 1 are parallel?

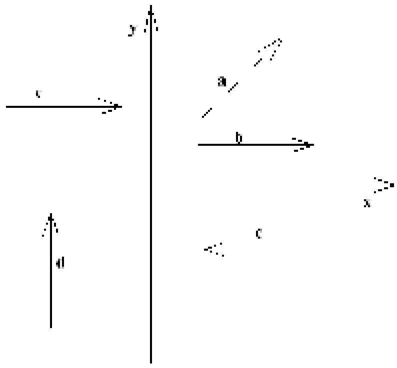


**Fig. 1**

a) **a, b**

b) **b, c**

c) **b, e**

d) **d, e**

**The second level**

What is the value of the expression **a**-2**b**-3**c**, if $a = \{-1; 2; 3\}, b = \{3; 2; 6\}, c = \{0, 2, 3\}$ ?

a) {-7;-8;-18};

b) {-14;-13;-13};

c) {5;12;24}.

d) none of the above.

**The third level**

Find the height of the parallelepiped $ABCDA'B'C'D'$ the base of which is the parallelogram $ABCD$, if $\overline{AB} \in a = (-1; 3; 4)$, $\overline{AD} \in b = (5, 0, 1)$ and $\overline{AA'} \in c = (2; -1; -4)$

a) $\sqrt{3}$;

b) 45;

c) $15\sqrt{3}$;

d) none of the above.

**The forth level**

Prove that the centers of gravity of all of the four faces of a tetrahedron are vertices of a tetrahedron, which is homothetic to the given.

*2. Different approaches to the problem statement, which would characterize the concept from different points of view.*

In this way, the problem is introduced in various contexts and in relationship with other concepts. For example, the concept of the "direction vector" can be represented as follows:

**1) A line in a plane can be uniquely defined by:**

a) a direction vector;

b) two non coplanar vectors;

c) a point and a direction vector;

d) a point and two non coplanar vectors.

**2) How many direction vectors can a line have?**

a) only one;

b) only two;

c) only three;

g) only four;

e) an infinite set.

**3) How is the direction vector positioned with respect to the line?**

a) at $90^0$ angle

b) at any angle;

c) parallel to the line.

**4) Can the direction vector belong to the line?**

a) yes;

b) no.

This approach allows the assessment of the depth of the actual understanding of the concepts related to a particular subject matter, as well as student's ability to answer questions, which may seem quite simple at first glance. Moreover, the student learns to answer a lot of analogous, but fundamental questions and gets prepared to offer the material to his/her pupils.

3. *Using pictures for the problem statement.*

It goes without saying that math problems and, especially, geometry problems should employ graphic presentations, which provide for the development of students' spatial perception, teach them to see the correlation between the concepts and their graphical interpretations. It is also important that, using graphical representations, the instructor can modify the level of difficulty. For instance, for the 1st and the 2nd levels, the problem on the slope-intercept form of a straight line equation can be presented as follows:

**1) The equation of a straight line is $y = kx + b$. Which segment on Figure 2 corresponds to the value of parameter $b$?**

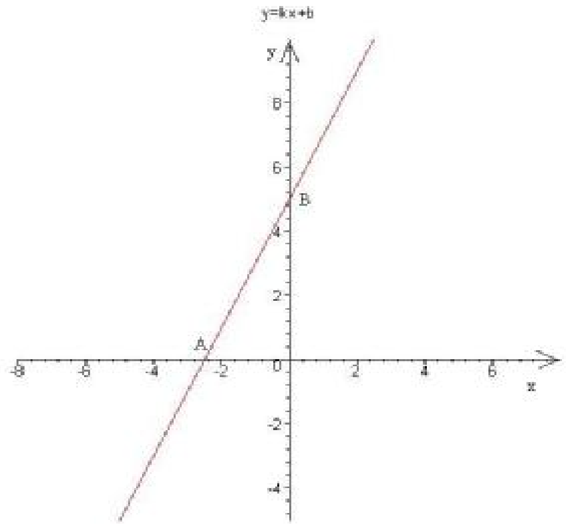



**Fig. 2**

a) AO; b) OB; c) AB.

**2) Which of the lines shown on Figure 3 has the largest slope value $k$?**

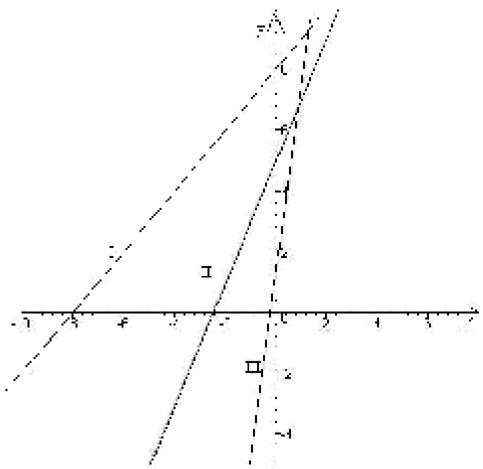


**Fig. 3**

a) I; b) II; c) III.

**3) What is the value of $tg\angle A$ for the line shown on Figure 4?**

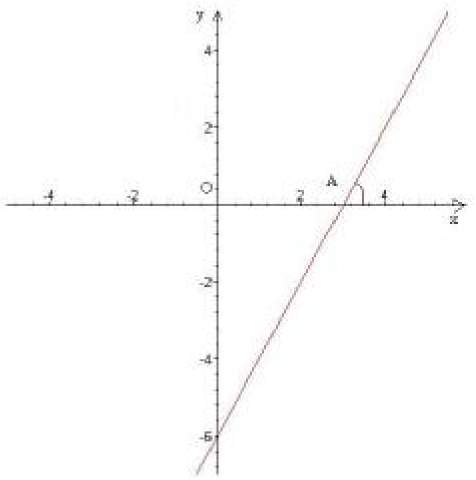


**Fig. 4**

a) 2; b) 3; c) -6; d) 0,5;

e) -0,5; f) -2.

The third-level problem for the slope-intercept form of a straight line equation is more complicated:

***Which of the proposed sets of points are equidistant from lines*** $l_1, l_2$***, if the slope of line*** $l_1$ ***is*** $3$ ***and it passes through the point*** $(0;1)$***, whereas line*** $l_2$ ***is shown on Figure 5:***

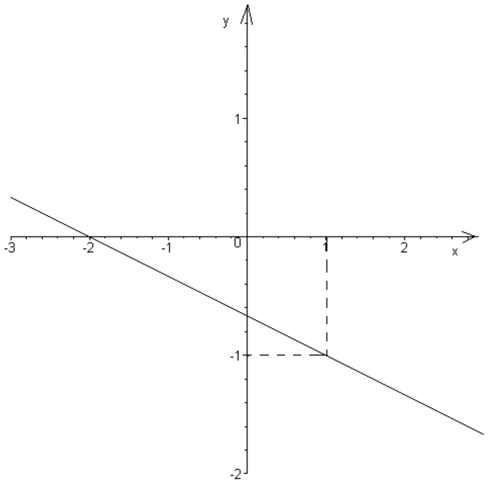



**Fig. 5**

a) $\left(\frac{1}{2};0\right);\left(-\frac{3}{4};0\right)$

b) $\left(-\frac{1}{2};0\right);\left(-\frac{3}{4};0\right)$

c) $\left(\frac{1}{2};0\right);\left(\frac{3}{4};0\right)$

d) none of the above.

*4. Variation in presenting test keys.*

In addition to different problem statements, the level of difficulty can be changed by the way the answers are presented.

For example, the options for the problem

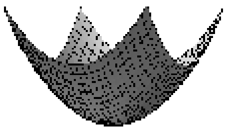
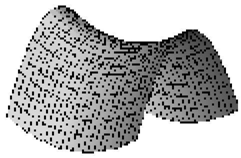
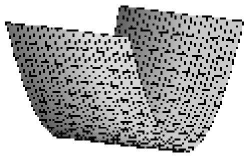
***1) What kind of surface is defined by the equation*** $z=\frac{x^2}{2p}-\frac{y^2}{2q}$***?***

a) elliptic paraboloid;

b) hyperbolic paraboloid;

c) parabolic cylinder.

can be presented in the following form:

a)  b)  c) 

This allows for the transition from the second to the third level and, at the same time, ensures the in-depth knowledge assessment.

Therefore, the use of a multi-level test structure allows the mathematics content knowledge assessment to begin from the high school curriculum. The latter is of particular importance for the training of prospective mathematics teachers, since it provides for building the link between the content of mathematical disciplines taught at secondary schools and in higher educational establishments. The prospective teacher of mathematics has to have a developed mathematical culture in order to be able to promote it in his/her own pupils later on. To do this requires the achievement of such level of understanding of mathematics that enables the student to see school mathematics in the context of fundamental mathematics. "...A set of certain philosophical, professionally relevant features and characteristics of a university graduate..." [2] is the goal of teachers' education, which is the object of our further research into the problem.

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**Димарський Я. М., Кривко Я. П. Тестування з аналітичної геометрії та шляхи його практичної реалізації**

Статтю присвячено принципам формування тестів з геометрії для студентів фізико-математичних педагогічних спеціальностей. Узагальнено переваги тестових завдань порівняно з класичними завданнями. Обґрунрунтовано необхідність створення тестів, максимально наближених до зовнішнього незалежного оцінювання. Запропоновано чотири рівні складності створюваних тестів з геометрії: 1) рівень елементарних понять, пропонованих у класичних термінах, 2) рівень елементарних понять, поданих у “нових” термінах, 3) базовий рівень виша, 4) високий рівень. Перші два рівні визначаються знаннями шкільного курсу з геометрії, третій й четвертий – стандартизованою програмою вищої школи. Завдання першого – третього рівнів побудовані за допомогою запитань закритого виду, на відповідність та на правильну послідовність. Для задач четвертого рівня можливі також завдання відкритого виду, що перевіряє безпосередньо викладач.

*Ключові слова:* тести, контроль навчальних досягнень, математична освіта, рівні складності тестів, зовнішнє незалежне тестування, аналітична геометрія.

**Димарский Я. М., Кривко Я. П. Тестирование по аналитической геометрии и пути его практической реализации**

Статья посвящена принципам формирования тестов по геометрии для студентов физико-математических педагогических специальностей. Обобщены преимущества тестовых заданий по сравнению с классическими задачами. Обоснована необходимость создания тестов, максимально приближенных к внешнему независимому оцениванию. Предложены четыре уровня сложности создаваемых тестов по геометрии: 1) уровень элементарных понятий, предлагаемых в классических терминах; 2) уровень элементарных понятий, представленых в “новых” терминах; 3) базовый уровень вуза; 4) высокий уровень. Первые два уровня определяются знаниями школьного курса геометрии, третий и четвертый – стандартизированной программой высшей школы. Задачи первого – третьего уровней строятся с помощью вопросов закрытого типа, на соответствие и на правильную последовательность. Для задач четвертого уровня возможны также задания открытого вида, которые проверяются непосредственно преподавателем.

*Ключевые слова:* тесты, контроль учебных достижений, математическое образование, уровни сложности тестов, внешнее независимое тестирование, аналитическая геометрия.

**Dymars’ky Ya. M., Kryvko Ya. P. Analytical Geometry Testing and the Ways of its Practical Implementation**

The article discusses the principles of developing tests in geometry for students working toward their degree as teachers of Physics and Mathematics. The advantages of tests compared to other forms of assessment are discussed. The need to develop tests similar to the ones included in the External Independent Assessment is demonstrated. It is suggested that tests in geometry should be of four levels of difficulty: 1) the level of elementary notions given in the form of classical terms; 2) the level of elementary notions presented with the help of “new” terms; 3) the basic level of a higher educational institution; and 4) the advanced level. The author introduces the principles that should be taken into consideration while developing tests in geometry for each level. The first two levels are aimed at testing the knowledge of school geometry course, whereas the third and the fourth are tailored for the standardized curriculum of higher educational institutions. The students’ progression from one level to another is viewed as the indication of the acquisition by them of the content knowledge, as well as their preparedness to teach this content in secondary schools. The task of the first through third levels are close-ended, or multiple-choice, questions, including matching and correct sequence items, while at the fourth level open-ended questions to be assessed by the instructor are used.

*Key words:* tests, monitoring of learning achievements, mathematical education, levels of difficulty of the tests, External Independent Assessment, analytical geometry.

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